1. What matrix has the effect of rotating every vector through 60° and then projecting the result onto the y-axis?

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\theta = 60°$

$$\begin{bmatrix} 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{bmatrix}$$

2. What matrix represents projection onto the y-axis followed by reflection over the x-axis?

Projection:

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

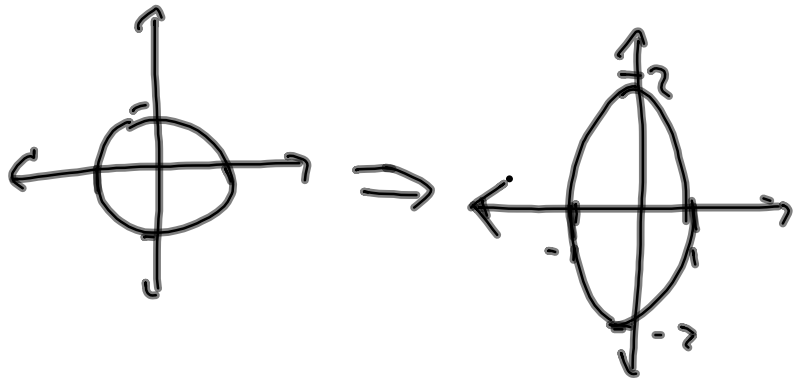
Reflection:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

3. What matrix produces a stretching in the y-direction by a factor of 2? Then, Draw the circle $x^2 + y^2 = 1$ and its result from multiplication by the matrix from the previous part.

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$



4. State the matrix for the given transformation and its result.
   a. (1, 2) after rotation by 30°
   b. (3, -2) after reflection over $y = 2x$

a.)

$$\begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} =$$

$$\begin{bmatrix} \cos 30 - 2\sin 30 \\ 1\sin 30 + 2\cos 30 \end{bmatrix}$$

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\theta = \tan^{-1} 2$$

$$\begin{bmatrix} 3\cos 2(\tan^{-1}(2)) - 2\sin 2(\tan^{-1}(2)) \\ 3\sin 2(\tan^{-1}(2)) + 2\cos 2(\tan^{-1}(2)) \end{bmatrix}$$

5. What 3 by 3 matrices represent the transformations that
   a. project every vector onto the *x*-*y* plane?
   b. reflect every vector through the *x*-*y* plane?
   c. rotate the *x*-*y* plane through 90°, leaving the *z*-axis alone?

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

c) $\begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. Suppose a linear T transforms (1, 1) to (2, 2) and (2, 0) to (0, 0). Find $T(v)$ when (a) $v = (2,2)$. (b) $v = (3,1)$. (c) $v = (-1,1)$. (d) $v = (a, b)$.

a) $\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ | $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$a+b=2$ $\quad a+d=2$ | $2a=0$ $\quad 2c=0$

$a=c=0$

$\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$

7. (a) What matrix transforms (1, 0) into (1, 5) and transforms (0,1) to (2, 2)?
(b) What matrix transforms (1, 5) to (1,0) and (2, 2) to (0,1)?
(c) Why does no matrix transform (1, 5) to (1,0) and (2, 10) to (0,1)?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$a = 1 \quad b = 2$

$c = 5 \quad d = 2$

$$\begin{bmatrix} 1 & 2 \\ 5 & 2 \end{bmatrix}$$

b)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$a + 5b = 1$
$c + 5d = 0$

$2a + 2b = 0$
$2c + 2d = 1$

$a + \frac{5}{4} = 1$

$2c + 10d = 0$

$8d = 1$

$d = -1 \quad c = 5$

$a = 1 - 5b$

$2 - 10b + 2b = 0$

$-8b = -2$

$b = \frac{1}{4}$

$a = -\frac{1}{4}$